

Analysis of an irreversible Ericsson engine with a realistic regenerator

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Abstract

An internally irreversible Ericsson engine, with a realistic regenerator, has been analyzed. The study considers internal irreversibilities with the introduction of turbine and compressor thermal-efficiencies and pressure-drops present in realistic regenerators. The effects of internal irreversibilities on the power output and thermal efficiency of the cycle have been determined using the finite-time thermodynamics. Maximum power-density, rather than maximum power, was used as the criterion for optimization, with the objective of having a more efficient small-sized engine. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

The heat-transfer processes in real heat engines must occur across finite-temperature differences in a limited time. This leads to irreversible heat transfers between the heat engine and its reservoirs. There are also other irreversibilities due to fluid friction, pressure drops, heat leaks, and so on. Thus it is necessary to investigate more comprehensively the influence of finite-rate heat-transfer, together with other major irreversibilities, on the performances of heat engines [1]. The analysis of irreversibility of finite-rate external heat-transfer processes of cycles constitutes a broad category of finite-time thermodynamics; namely, the endoreversible cycles [2–7]. Internal irreversibilities have been studied either by using a single parameter representing the ratio of entropy productions according to the second law of thermodynamics [1,8,9], or by introducing thermal efficiencies of cycle components and pressure-drop parameters [10,11].

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Nomenclature

c_v	constant-volume specific heat ($\text{kJ kg}^{-1} \text{K}^{-1}$)
k	ratio of principal specific heats (c_p/c_v)
$\dot{m}$	mass-flow rate (kg s^{-1})
n	polytropic exponent, dimensionless
P	pressure (kPa)
$\dot{Q}_H$	rate of polytropic heat transfer from the high temperature heat source to the working fluid (kW)
R	universal gas constant ($\text{kJ kg}^{-1} \text{K}^{-1}$)
r_p	compressor's compression-ratio (P_2/P_1)
T	temperature (K)
$\dot{W}$	cycle power-output (kW)
$\dot{W}_d$	cycle power-density ($\dot{W}_{\text{net}}/v_4$)
$\dot{W}_{d_{\text{max}}}$	maximum power-density
$\dot{W}_{\text{max}}$	maximum power-output

Greeks

v	specific volume ($\text{m}^3 \text{kg}^{-1}$)
η	thermal efficiency, dimensionless
β	pressure-drop parameter, dimensionless
Ψ	temperature ratio (T_2/T_4), dimensionless

Subscripts

c	irreversible compressor
C	Carnot
$C-A$	Curzon–Ahlborn
d	density
h	hot stream
l	cold stream
max	maximum
net	net
rev	reversible
t	irreversible turbine

In studies of heat-engine performance, a special place is reserved for regenerative cycles due to the fact that the use of a regenerator increases the thermal efficiency. Regenerative heat engines have been investigated mostly under the condition of perfect regeneration [12,13]. In some studies, more realistic approaches have been taken to practical regenerators [14–16]. The optimum performances of a regenerative Brayton cycle coupled to infinite heat sources and also variable temperature sources have been analyzed by considering external and internal irreversibilities [17,18].

Considering the fact that none of the previous investigations on the regenerative Ericsson engine included the analysis of internal irreversibilities, an irreversible

Ericsson heat engine is examined in this study, allowing for imperfect efficiencies of the turbine and compressor and pressure drop in the regenerator. The regenerator is also treated as a realistic one in which there is a finite temperature-difference due to polytropic expansion and compression processes in the turbine and compressor, respectively [16]. In addition, the irreversible Ericsson heat-engine is optimized using the maximum-power-density technique to introduce the engine size into the analysis in order to obtain a more efficient and small sized engine in comparison with that found when searching simply for the maximum-power condition.

2. Theory and mathematical model

A steady-state turbocompressor and T-s diagram for the irreversible Ericsson cycle are shown in Fig. 1(a) and (b), respectively. The process between points 1 and 2 is a polytropic compression in the compressor. The expansion in the turbine is represented by a polytropic curve between points 3 and 4. These paths correspond to reversible processes. In operation, since the compressor and turbine processes are high-flow rate processes and involve work-related devices, the isothermal processes deviate [19]. Therefore, we consider polytropic processes instead of isotherms. In order to introduce irreversibilities of the power components into the analysis, thermal efficiencies η_t and η_c are introduced for the turbine and compressor, respectively: η_t and η_c are defined as

$$\eta_t = \frac{\dot{W}_t}{\dot{W}_{trev}} \quad (1)$$

$$\eta_c = \frac{\dot{W}_{crev}}{\dot{W}_c} \quad (2)$$

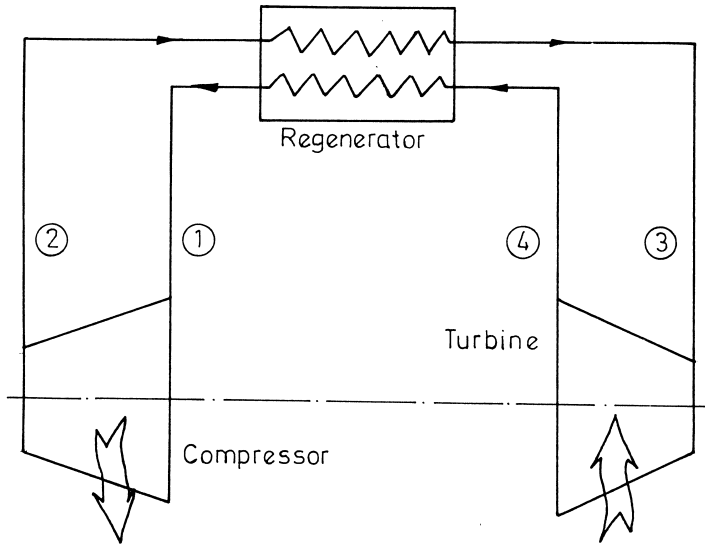
Considering the polytropic exponents n_t for the turbine and n_c for the compressor, the real turbine power-output and the power requirement of the compressor, for an ideal-gas working fluid become, respectively

$$\dot{W}_t = \dot{m}R \frac{n_t}{1 - n_t} \eta_t (T_4 - T_3) \quad (3)$$

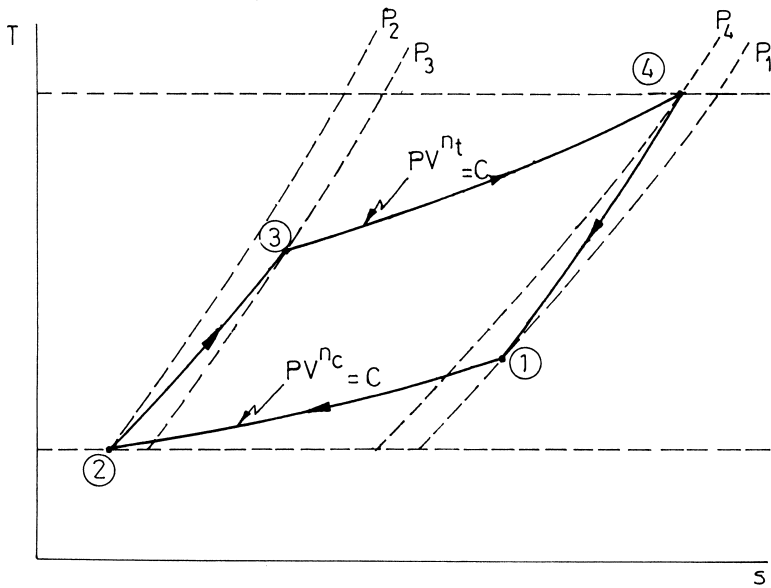
$$\dot{W}_c = \dot{m}R \frac{n_c}{1 - n_c} \frac{1}{\eta_c} (T_1 - T_2) \quad (4)$$

Due to these polytropic processes, a finite temperature-difference exists between the hot and cold streams. To ensure heat transfer in the regenerator, polytropic exponents for turbine n_t and compressor n_c should have values such that $0 < n_t < 1$ and $0 < n_c < 1$.

Irreversibilities in the regenerator are expressed in terms of pressure drops. Using the symbols P_{in} and P_{out} for the inlet and outlet pressures, respectively, of a stream [20], a pressure-drop parameter is defined by



(a)



(b)

Fig. 1. (a) A steady-flow turbo-compressor and (b) a T-s diagram for the Ericsson cycle.

$$\beta = \frac{P_{\text{out}}}{P_{\text{in}}} = 1 - \frac{P_{\text{in}} - P_{\text{out}}}{P_{\text{in}}} = 1 - \frac{\Delta P}{P} \quad (5)$$

Then,

$$\beta_h = \frac{P_1}{P_4} \text{ and } \beta_l = \frac{P_3}{P_2} \quad (6)$$

represent the pressure-drop parameters for the hot and cold streams in the regenerator, respectively. The irreversibility parameters η_t , η_c , β_h and β_l are all less than unity. Cycle temperature ratios at the ends of each branch can be defined in terms of the compressor compression ratio $r_p (= P_2/P_1)$, temperature ratio $\Psi (= T_2/T_4)$, and pressure-drop parameters β_h and β_l , as follows:

$$\frac{T_2}{T_1} = r_p^{\frac{n_c-1}{n_c}} \quad (7)$$

$$\frac{T_3}{T_4} = (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \quad (8)$$

$$\frac{T_1}{T_4} = \Psi r_p^{\frac{1-n_c}{n_c}} \quad (9)$$

$$\frac{T_3}{T_2} = \frac{1}{\Psi} (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \quad (10)$$

The net power of the cycle becomes

$$\dot{W}_{\text{net}} = \dot{m} R T_4 \left\{ \frac{n_t}{n_t - 1} \eta_t \left(1 - (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \right) - \frac{n_c}{1 - n_c} \frac{\Psi}{\eta_c} \left(r_p^{\frac{1-n_c}{n_c}} - 1 \right) \right\} \quad (11)$$

In the regenerator, the amounts of heat transferred from the hot stream to the cold one should be equal, then the regeneration condition is obtained as follows.

$$\Psi = \frac{\eta_t \left[1 - (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \right]}{\frac{1}{\eta_c} \left[r_p^{\left(\frac{1-n_c}{n_c} \right)} - 1 \right]} \quad (12)$$

From Eq. (12), n_c can be expressed in terms of r_p , Ψ , n_t , η_t , η_c , β_h and β_l , as

$$n_c = \frac{\ln r_p}{\ln r_p + \ln \left[1 + \frac{\eta_t \eta_c}{\Psi} \left(1 - (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \right) \right]} \quad (13)$$

Using Eq. (13), the specific net power of the cycle becomes

$$\frac{\dot{W}_{\text{net}}}{\dot{m}RT_4} = \eta_t \left(1 - (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \right) \Phi \quad (14)$$

where

$$\Phi = \left(\frac{n_t}{1 - n_t} - \frac{n_c(\Psi, n_t, r_p, \eta_t, \eta_c, \beta_h, \beta_l)}{1 - n_c(\Psi, n_t, r_p, \eta_t, \eta_c, \beta_h, \beta_l)} \right) \quad (15)$$

It is seen from Eqs. (14) and (15) that n_t should have a value $> n_c(\Psi, n_t, r_p, \eta_t, \eta_c, \beta_h, \beta_l)$ to obtain a positive net power. The power density of the cycle is found by dividing the net power output of the cycle by the maximum specific volume during the cycle (V_4) as follows

$$\dot{W}_d = \frac{\dot{W}_{\text{net}}}{V_4} = \frac{\dot{m}RT}{V_2} \frac{\eta_t}{\beta_h r_p} \left(1 - (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \right) \Phi \quad (16)$$

The specific heat transferred to the working fluid at the turbine is given by

$$\dot{Q}_H = \dot{Q}_{34} = \dot{m}c_v T_4 \frac{(k - n_t)}{(1 - n_t)} \eta_t \left(1 - (\beta_h \beta_l r_p)^{\frac{n_t-1}{n_t}} \right) \quad (17)$$

The thermal efficiency of the cycle is obtained from Eqs. (14) and (17) as

$$\eta = \frac{(k - 1) [n_t - n_c(\Psi, n_t, r_p, \eta_t, \eta_c, \beta_h, \beta_l)]}{(k - n_t) [1 - n_c(\Psi, n_t, r_p, \eta_t, \eta_c, \beta_h, \beta_l)]} \quad (18)$$

3. Results

The effects of irreversibilities on the Ericsson cycle were investigated and variations of normalized power and power density, efficiency and compression ratio were obtained under different prescribed conditions. The maximization of power has been accomplished numerically by setting the first derivative of power with respect to compression ratio equal to zero. This leads to the optimized compression ratio, which is used in the power to get the maximized power at the prescribed conditions. The same procedure is followed for maximizing the power density. Fig. 2 shows the changes in thermal efficiency obtained at maximum power-density and at maximum-power conditions, with changes in the turbine and compressor efficiencies. When Fig. 2(a) and (b) are compared, it is seen that the values of the thermal efficiency at maximum power-density design condition are higher than the values of the thermal efficiency at maximum-power condition for all the turbine and compressor efficiencies. Both the thermal efficiency at maximum-power density and the thermal efficiency at maximum power are sensitive to the inefficiencies of the turbine and compressor. Especially at low values of the turbine and compressor efficiencies, increasing sensitivity is clearly observed.

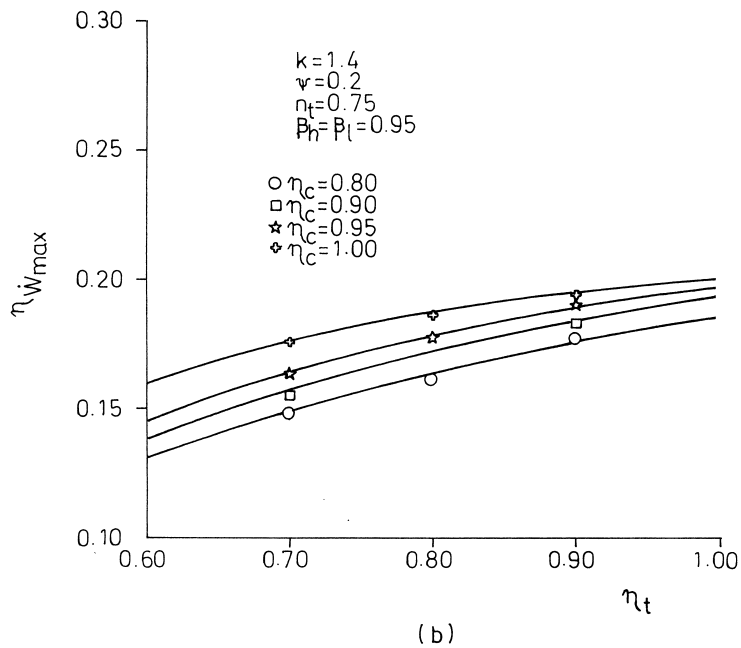
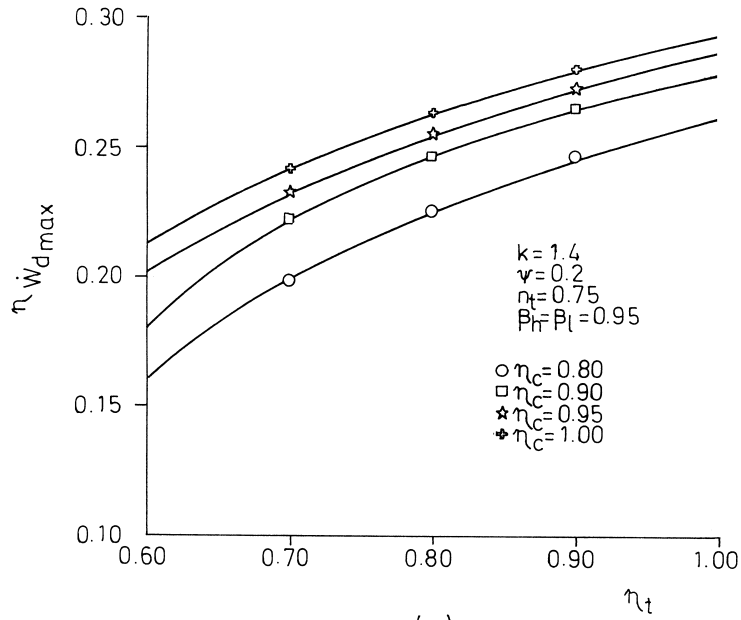


Fig. 2. Effects of turbine and compressor thermal efficiencies on the cycle efficiency at (a) maximum-power density and (b) maximum power.

The values of normalized power and normalized power-density were calculated at different values of turbine and compressor efficiencies. The results are shown in Fig. 3 as functions of cycle thermal efficiency. When Fig. 3(a) and (b) are compared, the following is observed. For all values of turbine and compressor efficiencies, cycle efficiencies are higher at the maximum power density design condition. As turbine and compressor efficiencies increase, i.e. turbine and compressor have smaller

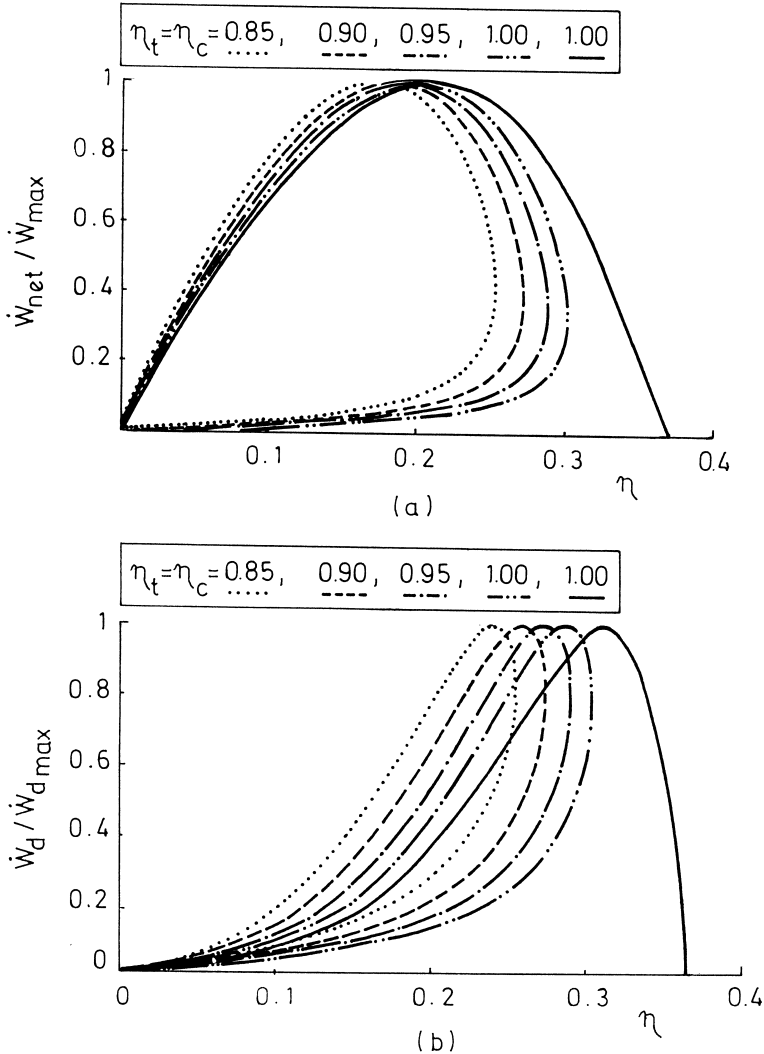


Fig. 3. The variations of (a) normalized power-density and (b) normalized power according to thermal efficiency with the consideration of the effects of turbine and compressor efficiencies ($k = 1.4$, $\Psi = 0.2$, $n_t = 0.75$, $\beta_h = \beta_l = 0.95$ and $\beta_h = \beta_l = 1.0$ for the solid lines).

irreversibilities, the cycle thermal efficiency at the maximum power-density design condition [Fig. 3(a)] shows a rapid response and reaches higher values than those achieved at the maximum-power condition. The range of variation in the cycle thermal efficiency for designs considering maximum-power conditions lies within a narrow region [Fig. 3(b)] while the change in turbine and compressor efficiencies is from

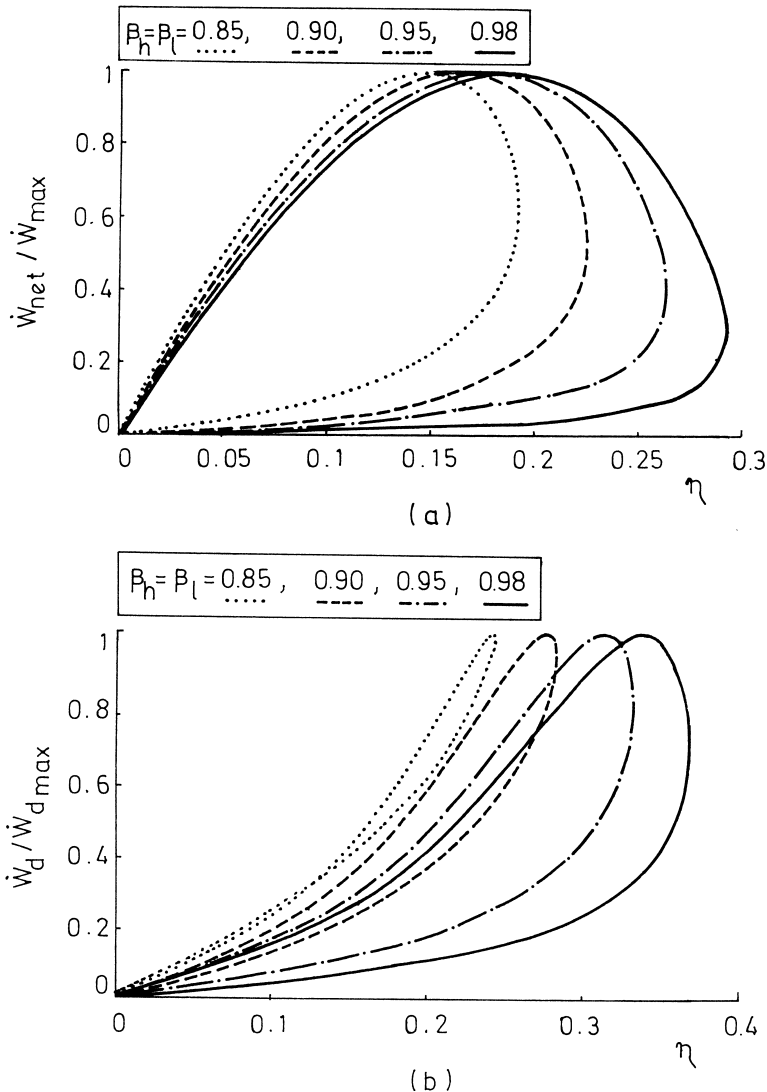
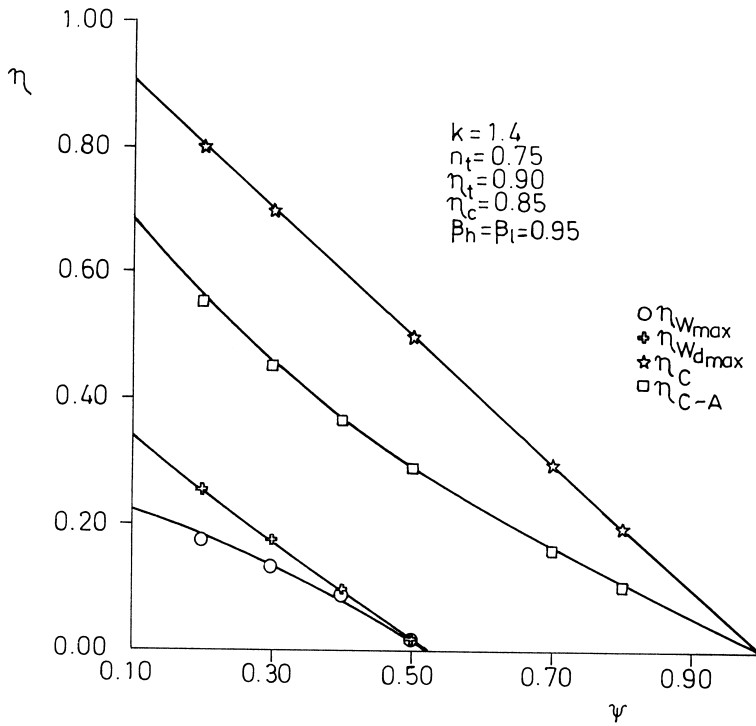
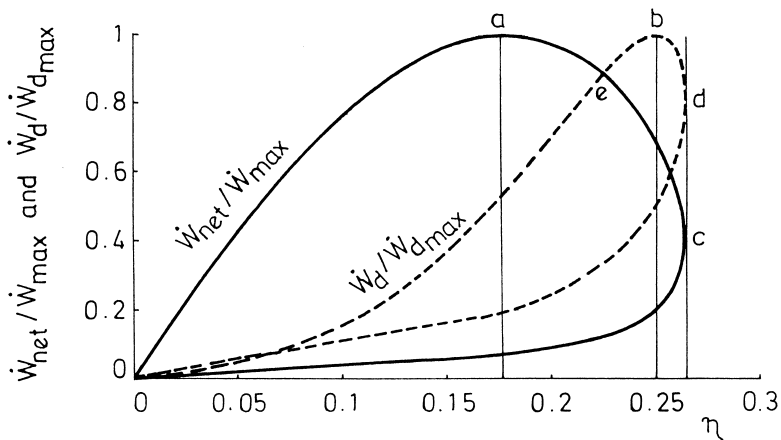


Fig. 4. The effect of pressure drops in the regenerator on the variations of (a) normalized power-density and (b) normalized power with respect to thermal efficiency ($k = 1.4$, $\Psi = 0.2$, $n_t = 0.75$, $\eta_t = 0.90$, $\eta_c = 0.85$).

Fig. 5. Comparison of efficiencies with temperature ratio, Ψ .Fig. 6. The comparison of normalized power ($\dot{W}_{\text{net}}/\dot{W}_{\text{max}}$) and normalized power-density ($\dot{W}_d/\dot{W}_{d_{\text{max}}}$) according to thermal efficiency ($k=1.4$, $\Psi=0.2$, $\eta_t=0.75$, $\eta_t=0.95$, $\eta_c=0.90$, $\beta_h=\beta_l=0.90$).

0.85 to 1.0. In other words, designs considering the maximum power condition have weak responses to increasing efficiency of the power components.

The effects of irreversibilities during heat transfer in the regenerator on the performance of the Ericsson cycle are summarized in Fig. 4(a) and (b), using the pressure-drop parameters for the hot and cold streams in the regenerator, β_h and β_l . At the same values of the pressure-drop parameters, thermal efficiencies at normalized power density values are always greater than that of normalized power values. As the pressure-drop parameters approach values near the reversible case, the efficiencies increase in both cases, but the rate of increase is higher for maximum power-density designs. The rapid responses to incremental improvements establish the powerful aspect of designs based upon the maximum power-density design condition.

Fig. 5 shows the effect of temperature in terms of temperature ratio at prescribed values of the polytropic exponent for turbine η_t , turbine and compressor efficiencies,

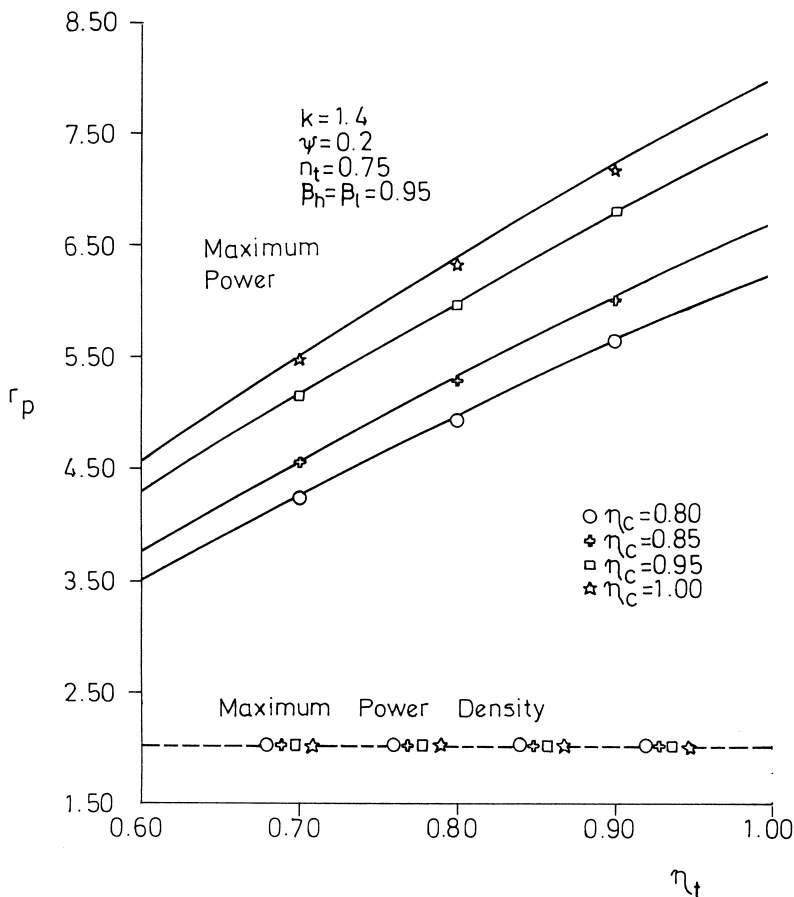


Fig. 7. Variation of compressor compression ratio at maximum power and maximum power-density with the compressor and turbine thermal-efficiencies.

and the pressure-drop parameters for the hot and cold streams in the regenerator, for engines designed for the maximum power and the maximum power-density design conditions. Especially at small temperature ratios, the values of the thermal efficiency at maximum power-density are higher than the values of the thermal efficiency at maximum power. Curzon–Ahlborn and Carnot efficiencies (η_{C-A} and η_C , respectively) are also shown for comparison.

To obtain a detailed analysis, the values of normalized power and normalized power-density are compared, as shown in Fig. 6. Points (a) and (b) represent the values of the thermal efficiency at maximum power and the thermal efficiency at maximum power-density design conditions, respectively. Fig. 6 shows that the thermal efficiency at maximum power-density is approximately eight percent higher than the thermal efficiency at maximum power under the conditions chosen. Points (d) and (c) correspond to maximum cycle-efficiency. The point (e) at the intersection of the two curves has a special meaning. Point (e) has an efficiency between the thermal efficiencies at maximum power and maximum power-density design conditions, that is $\eta_{\dot{W}_{\max}} < \eta_e < \eta_{\dot{W}_{\text{dmx}}}$, and carries the advantages of both design criteria. In fact, the point (e) is in good agreement with the recent study of De Vos [21] who stated that the optimal point lies between the points of maximum power and maximum efficiency.

The most important result of the present study can be seen in Fig. 7, which shows the variation of compressor compression ratio (r_p) at the maximum power and the maximum power-density design conditions, as functions of turbine and compressor efficiencies values. At the maximum power condition, the values of compressor compression ratio are well above those at the maximum power-density design condition. When the engine has large irreversibilities, the values of compressor compression-ratio at the maximum-power condition decrease considerably, which in turn results in a decrease in power. On the other hand, the values of compressor compression-ratio at the maximum power-density design condition remain nearly constant, meaning that compact compressor size will be obtained at the maximum power-density design condition. The maximum power-density analysis for a regenerative Joule–Brayton cycle has shown that the irreversible gas-turbine is also smaller under the maximum-density conditions [22].

4. Conclusion

The purpose of this study has been to examine the effect of irreversibilities on the performance of the Ericsson cycle. Irreversibility parameters were introduced as thermal efficiencies of the power producing and consuming components and of the pressure-drop parameters for the regenerative heat-transfer unit.

The cycle power and power density are decreased due to the inefficiencies of the turbine and compressor. The designs made subject to the maximum power-density design conditions have better responses to increasing efficiencies of the power components. When the irreversibilities in the regenerator were decreased, the engine designed according to the maximum power-density design condition gave better performance and exhibited higher cycle efficiencies than the others.

In all cases, the compressor compression-ratios under the maximum power-density design conditions have an optimum value independent of the size of the compressor. Therefore compressors designed under the conditions of the maximum power-density can be more compact. The maximum power-density analysis applied to the irreversible Ericsson cycle, with realistic regenerator, results in small and efficient engine designs.

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